

# Mitigating destabilization effects due to line dynamics in multi-inverter systems

Guru Praanesh Raman

NUS

# Outline

- 1 Why generalized droop?
- 2 Multi-inverter system
  - Small-signal analysis with static line flow
  - Small-signal analysis with dynamic line flow
- 3 EM-induced phenomena
  - Distribution System Lag Factor (DSLFF)
  - EM-induced damping
  - EM-induced cross-coupling
- 4 Proposed filter design
- 5 Simulation results
- 6 Conclusions

# Why generalized droop?

Conventional droop equation:

$$\begin{bmatrix} \Delta f \\ \Delta V \end{bmatrix} = \begin{bmatrix} k_f & 0 \\ 0 & k_v \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (1)$$

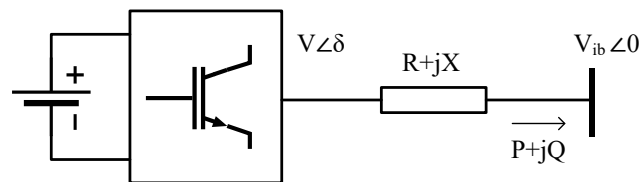


Figure 1: Inverter connected to an infinite bus through  $Z \angle \theta = R + jX$ .

## General power flow equations

$$P = \frac{VV_{ib}}{Z} (\sin \theta) \delta + \frac{V_{ib}}{Z} (\cos \theta) (V - V_{ib}) \quad (2)$$

$$Q = \frac{VV_{ib}}{Z} (\cos \theta) \delta + \frac{V_{ib}}{Z} (\sin \theta) (V - V_{ib}) \quad (3)$$

# The generalized droop law

Generalized droop equation:

$$\begin{bmatrix} \Delta f \\ \Delta V \end{bmatrix} = \begin{bmatrix} k_f \sin \theta & k_f \cos \theta \\ k_v \cos \theta & k_v \sin \theta \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (4)$$

## General power flow equations

$$P = \frac{VV_{ib}}{Z} (\sin \theta) \delta + \frac{V_{ib}}{Z} (\cos \theta) (V - V_{ib}) \quad (5)$$

$$Q = \frac{VV_{ib}}{Z} (\cos \theta) \delta + \frac{V_{ib}}{Z} (\sin \theta) (V - V_{ib}) \quad (6)$$

The droop occurs in a rotated power frame  $[\Delta \hat{P}, \Delta \hat{Q}] = \mathbf{T} [\Delta P, \Delta Q]$ , with:

$$\mathbf{T} = \begin{bmatrix} \cos \phi & \sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \quad \mathbf{I}_n \text{ and } \mathbf{T} \begin{bmatrix} \mathbf{B} & \mathbf{G} \\ \mathbf{G} & \mathbf{B} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{B}} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{B}} \end{bmatrix}$$

Here,  $\phi = 0.5\pi - \theta$  and  $\mathbf{I}_n$  is the identity matrix.

# Control Schematic

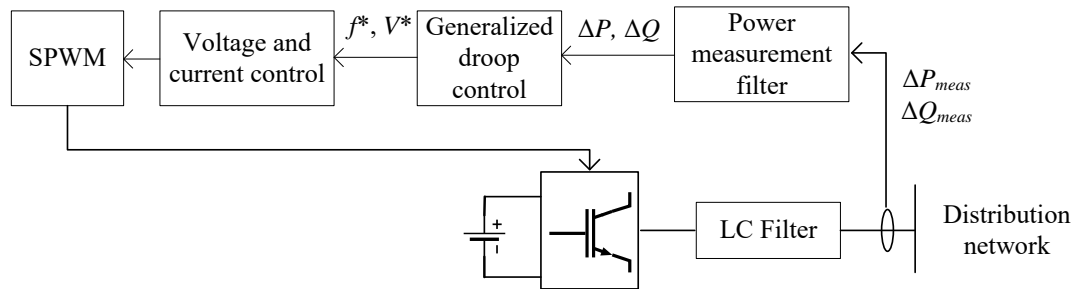


Figure 2: Control schematic of an inverter with generalized droop.

- First, **use generalized droop** to remove instability mechanism due to cross-coupling
- Incorporate EM dynamics to **observe remaining instability factors**
- **Develop control modification(s)** to achieve guaranteed dynamics stability for all droops and  $R/X$  values

# Third-order model

The characteristic equation with static Jacobian is:

$$\left( \begin{bmatrix} s(T_c s + 1)\mathbf{\Lambda}_f & \mathbf{0} \\ \mathbf{0} & (T_c s + 1)\mathbf{\Lambda}_v \end{bmatrix} \mathbf{T} \begin{bmatrix} \mathbf{B} & \mathbf{G} \\ \mathbf{G} & \mathbf{B} \end{bmatrix} \right) \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \mathbf{0} \quad (7)$$

where  $\mathbf{\Lambda}_f = \mathbf{K}_f^{-1}$  and  $\mathbf{\Lambda}_v = \mathbf{K}_v^{-1}$ .

Illustration- the single-inverter case:

$$T_c s^2 + s - k_f \hat{B} = 0 \quad (8)$$

$$T_c s + 1 - k_v \hat{B} = 0 \quad (9)$$

The oscillatory poles are **always stable**:

$$s = \frac{1}{2T_c} \pm \frac{1}{2T_c} \sqrt{1 + 4k_f \hat{B} T_c} \quad (10)$$

The real pole  $s = (1 - k_v \hat{B})/T_c$  is **always stable** as well.

# Incorporating line dynamics

We now use dynamic power system model as:

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{B}(s) & \mathbf{G}(s) \\ \mathbf{G}(s) & \mathbf{B}(s) \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \Delta \mathbf{V} \end{bmatrix}, \quad (11)$$

with the substitutions:

$$\begin{aligned} \mathbf{G}(s) &= \frac{(\rho^2 + 1)(\rho + s/\omega_0)}{(\rho + s/\omega_0)^2 + 1} \mathbf{B} \\ \mathbf{B}(s) &= \frac{(\rho^2 + 1)}{(\rho + s/\omega_0)^2 + 1} \mathbf{B} \end{aligned} \quad (12)$$

Here,  $\rho$  denotes the  $R/X$  ratio of the lines and  $\omega_0$ , the power frequency  $100\pi$  rad/s.



# Fifth-order model

With EM dynamics, the model may be obtained as:

$$\left( \underbrace{\frac{(\rho + s/\omega_0)^2 + 1}{(\rho^2 + 1)}}_{DSLF} \begin{bmatrix} s(T_c s + 1)\mathbf{\Lambda}_f & \mathbf{0} \\ \mathbf{0} & (T_c s + 1)\mathbf{\Lambda}_v \end{bmatrix} \begin{bmatrix} \hat{\mathbf{B}} & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{B}} \end{bmatrix} + \underbrace{\frac{s}{\omega_0} \begin{bmatrix} \mathbf{B} \sin \phi & \mathbf{B} \cos \phi \\ \mathbf{B} \cos \phi & \mathbf{B} \sin \phi \end{bmatrix}}_{\text{non-decoupled}} \right) \begin{bmatrix} \Delta \delta \\ \Delta \mathbf{V} \end{bmatrix} = \mathbf{0} \quad (13)$$

Phenomena to analyze:

- Distribution System Lag Factor (DSLF)
- EM-induced cross-coupling ( $\cos \phi$  terms)
- EM-induced damping ( $\sin \phi$  terms)

# Distribution System Lag Factor (DSLRF)

Temporarily neglecting cross-coupling term ( $\cos \phi$  term),

$$\left( \frac{s^2}{\omega_0^2} + \frac{2\rho}{\omega_0}s + 1 + \rho^2 \right) s(T_c s + 1) - (1 + \rho^2)k_f \hat{B} - \frac{(1 + \rho^2)B \sin \phi}{\omega_0} = 0$$

Using Routh-Hurwitz method, the stability condition is:

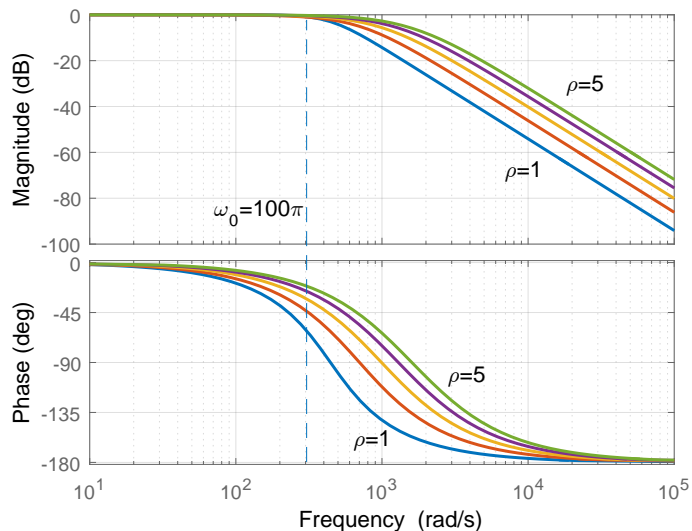
$$k_f < \frac{\omega_0(1 + \rho^2)}{B(1 + \rho^2) \sin \phi - 2\rho \hat{B}} \quad (14)$$

$\Rightarrow$  Stability is lost at higher  $k_f$ !

# Properties of DSLF

- Unity gain at zero frequency  
=> Does not change steady-state droop
- High lag at low  $R/X$  (1-3) and lower lag at higher  $R/X$  ( $>5$ )
- At high  $R/X$ ,  $DSL F \neq 1$

**Conclusion:** DSLF destabilizes system at lower  $R/X$  and has no effect at higher  $R/X$



**Figure 3:** Bode plot of  $1/DSL F$  for various  $R/X$  ratios indicating significant phase lag at power frequency  $\omega_0$ .

# EM-induced damping

Taking  $DSL F = 1$ , we obtain the characteristic equations as:

$$T_c s^2 + \left(1 + \frac{k_f B \sin \phi}{\omega_0}\right) s + k_f \hat{B} = 0 \quad (15)$$

$$\left(T_c + \frac{k_v B \sin \phi}{\omega_0}\right) s + 1 + k_v \hat{B} = 0 \quad (16)$$

- Since  $B$  and  $\hat{B}$  are both negative, system is always stable
- The  $\sin \phi$  terms increase damping, pushing poles to the left of  $\omega_c/2$

# EM-induced cross-coupling

- Cross-coupling appears due to  $\cos \phi$  terms
- This effect pushes poles rightward, but does not affect stability:

$$\left( T_c + \frac{k_f k_v B^2 \cos^2 \phi}{\omega_0^2 (1 - k_v \hat{B})} \right) s^2 + \left( 1 - \frac{k_f B \sin \phi}{\omega_0} \right) s - k_f \hat{B} = 0 \quad (17)$$

- Cross-coupling disappears at high R/X ratios as  $\cos \phi \rightarrow 0$

- DSLF is the key destabilization factor
- EM-induced damping improves damping
- EM-induced cross-coupling slightly reduces damping, but does not affect stability
- **Stability is guaranteed iff effect of DSLF is annulled, for all droops and R/X ratios**

# Proposed power filter

The initial design of the filter was:

$$F_0(s) = \frac{1}{T_c s + 1} \quad (18)$$

The proposed filter is:

$$F(s) = \frac{s^2/\omega_0^2 + 2\rho s/\omega_0 + 1 + \rho^2}{(\rho^2 + 1)(T_c s + 1)(\tau s + 1)} \quad (19)$$

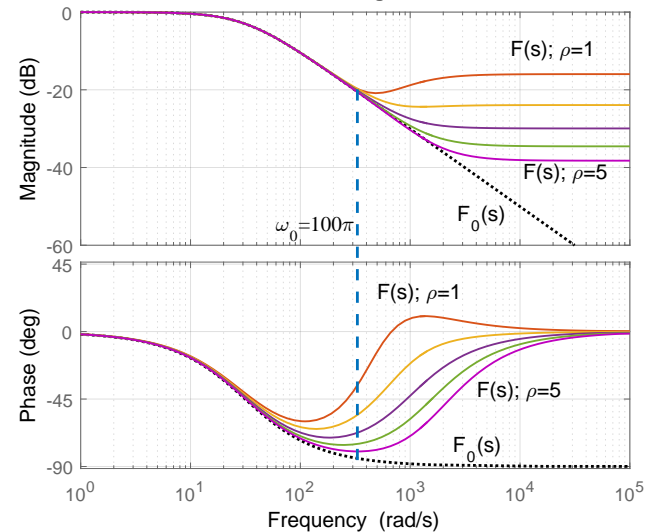


Figure 4: Bode plots for original filter  $F_0(s)$  and the proposed filter  $F(s)$  for various values of  $\rho$ . The proposed filter provides phase lead at  $\omega_0 = 314$  rad/s.

# Selection of $\tau$

- The dummy term  $(\tau s + 1)$  makes  $F(s)$  causal. We need  $\tau \ll T_c$ .
- The phase lead at  $\omega_0 = 314$  rad/s is higher for lower values of  $\tau$ .
- Higher values of  $\tau$  require high BW filter design. This is however unnecessary as  $\tau=1e-3$  is quite sufficient.

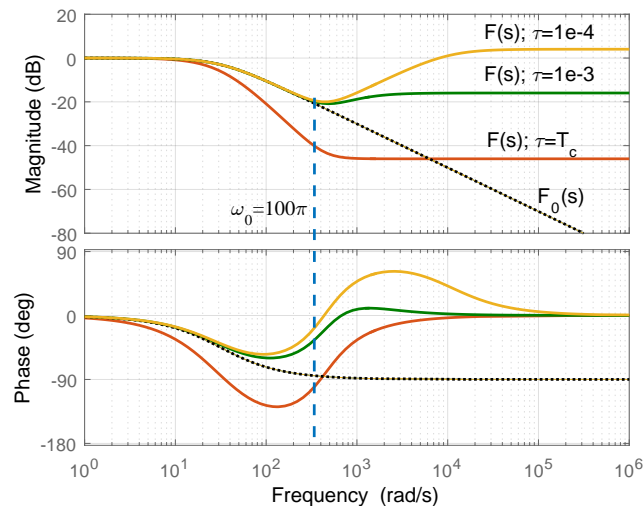


Figure 5: Bode plots for original filter  $F_0(s)$  and the proposed filter  $F(s)$  for various values of  $\tau$ .



# Closed loop poles

- The implementation of the proposed filter eliminates the effect of the DSLF
- The eigenvalues lie to the left of the vertical line at  $\omega_c/2$  rad/s.
- The poles do not lie exactly on that vertical line because of the EM-induced damping effect.

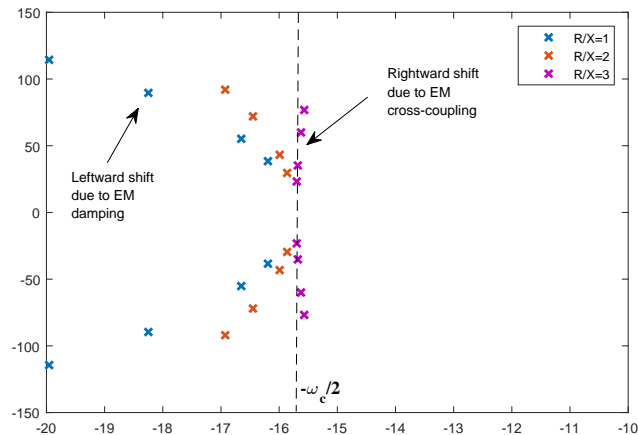


Figure 6: Closed loop poles for 5-inverter system illustrating discussed effects

# Time-domain results

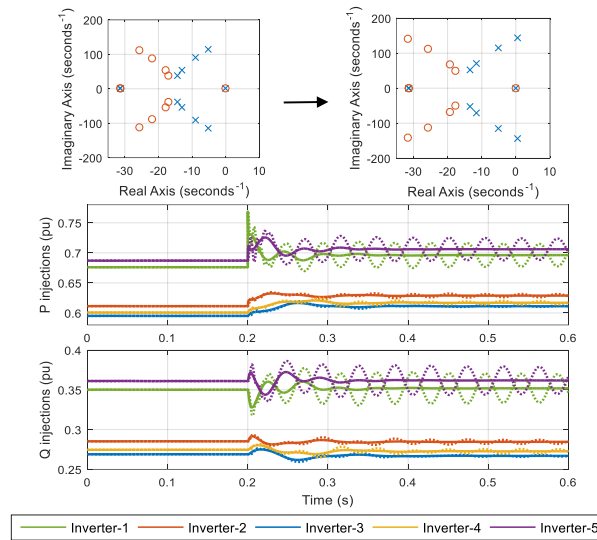
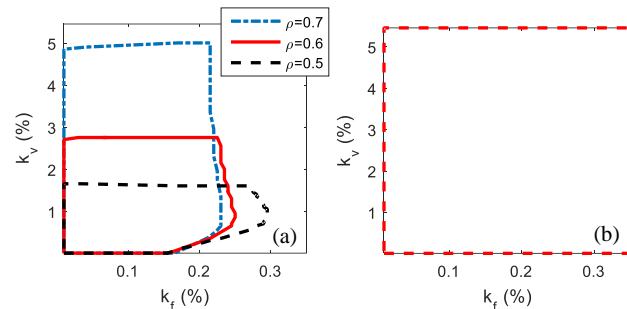


Figure 7: *Top*: Initial and final locations of poles for conventional filter (blue x) and proposed filter (red o). *Below*: Evolution of real and reactive power injections with the conventional filter (dashed lines) and proposed filter (solid lines) for the same droop settings, indicating well-damped behavior with the proposed filter.

# Stability region

- With the conventional filter, the stability region is smaller for smaller R/X values, and larger for larger R/X values.
- With the implementation of the proposed filter, the stability region becomes infinite.
- This reiterates that the DSLF is the sole instability factor to be addressed.



**Figure 8:** Stability region for generalized droop under various R/X ratios with (a) conventional filter (b) proposed filter (the entire plane is theoretically stable for (b)).

# Application to conventional droop

- Under conventional droop, both the instability phenomena: P-V/Q-f cross-coupling, and line dynamics exist.
- With the proposed filter, the stability region significantly expands for lower R/X values, and slightly for higher R/X.
- This implies that cross-coupling and line dynamics are *independent* instability phenomena, the former dominant at higher R/X values, and the latter at lower R/X.

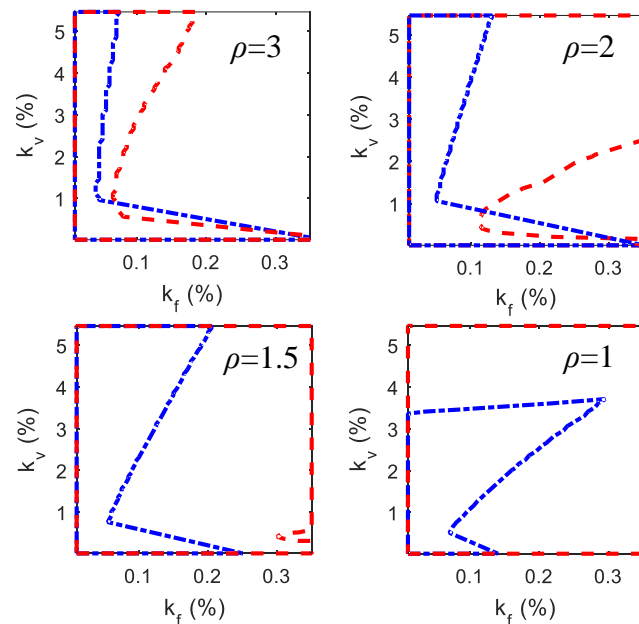


Figure 9: Stability region comparison for P-f/Q-V droop under various R/X ratios with conventional filter (blue dash-dotted lines) and proposed filter (red dashed lines) indicating

# Conclusions

- Line dynamics is an important stability in practical distribution systems with moderate or low R/X ratios ( $< 3$ )
- The proposed filter has been verified to fully eliminate this instability effect
- The proposed filter is also valid for systems with conventional droop control
- The design procedure is formula-based, independent of the system topology and operating point, and does not require any hardware modifications.